Design replication in partial-profile choice experiments

Heiko Großmann
OvGU Magdeburg, DE

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Outline

- Motivation
- Technical background
- Design replication
- Simulation study

Motivating example

Question 1

Which of the two explanations below would help you most to understand and manage your own osteoarthritis?

Explanation 1A	**Explanation 1B**
Osteoarthritis is the most common form of arthritis in the UK	You have osteoarthritis, a condition which can affect the whole joint and surrounding muscles
Pain, stiffness, and limitation in full movement of the joint are typical	Osteoarthritis can affect your joints in different ways at different times, sometimes you may not have any difficulties but at other times you might
You can take steps to improve your osteoarthritis, by being physically active, maintaining a healthy weight and thinking positively. This can help how you feel and what you can do now, and may help to avoid the need for more treatments in future. Support is available to help you to achieve this	There is no cure for osteoarthritis but there are a number of things that can be done to ease symptoms
Even a modest weight loss can make quite a difference	Even a modest weight loss can make quite a difference

Please tick one box only

Explanation 1A ☐ Explanation 1B ☐

R3-S253-Q2017-B2C01MDP163

Question 5

Which of the two explanations below would help you most to understand and manage your own osteoarthritis?

Explanation 5A	Explanation 5B
Osteoarthritis is caused by an ongoing process of wear and the joint trying to heal itself	Osteoarthritis occurs perhaps because of severe wear and tear to the joints or a problem with the repair process, and osteoarthritis develops
It is mild in many cases; however, about 1 in 10 people aged over 65 years have a major disability due to osteoarthritis	It is mild in many cases; however, about 1 in 10 people aged over 65 years have a major disability due to osteoarthritis
Many people can manage a regular walk	It can be easier to keep moving if you build up from where you are now and put new activities to improve your osteoarthritis in to your daily routine
Keeping active and maintaining a healthy weight are best for your osteoarthritis in the long run, even though some social activities can make this difficult	Many people are afraid to exercise because they believe, mistakenly, that it'll cause further damage to their joints

Please tick one box only

Explanation 5A ☐ Explanation 5B ☐

R1-S022-Q0173-B1C10MDP170

Partial profiles

Full profiles ...

... use **all** factors

A2	A1
B1	B2
C2	C1
D1	D1

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Partial profiles ...

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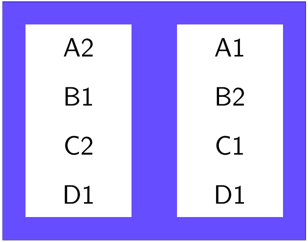

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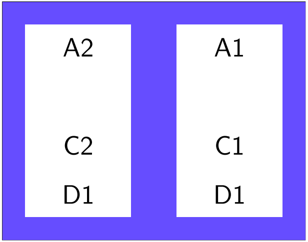

Profile strength S: Number of factors shown

Comparison depth d: Number of shown factors where alternatives differ

MNL model

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K factors with levels $1, \ldots, v_k$, $k = 1, \ldots, K$, of interest
and extra level 0 to indicate factor is not shown

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Pairs $(\mathbf{s}, \mathbf{t})$ of partial profiles where $\mathbf{s} = (s_1, \ldots, s_K)$, $\mathbf{s} = (t_1, \ldots, t_K)$
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MNL probability of choosing $\mathbf{s}$ from pair $(\mathbf{s}, \mathbf{t})$:

$$\boxed{P(\mathbf{s}; (\mathbf{s}, \mathbf{t})) = \frac{\exp[\mathbf{f}^\top(\mathbf{s})\boldsymbol{\beta}]}{\exp[\mathbf{f}^\top(\mathbf{s})\boldsymbol{\beta}] + \exp[\mathbf{f}^\top(\mathbf{t})\boldsymbol{\beta}]}}$$

Design efficiency

Exact partial-profile designs ξ_N: pairs $(\mathbf{s}_1, \mathbf{t}_1), \ldots, (\mathbf{s}_N, \mathbf{t}_N)$

Fisher information matrix of ξ_N in MNL Model: $\mathbf{M}_{\xi_N, \boldsymbol{\beta}}$

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- $\mathbf{M}_{\xi_N, \boldsymbol{\beta}} = \mathbf{M}_{\xi_N, \mathbf{0}} = \frac{1}{4}\mathbf{X}^\top \mathbf{X}$

  where $\mathbf{X}$ has rows $\mathbf{f}^\top(\mathbf{s}_n) - \mathbf{f}^\top(\mathbf{t}_n)$, $n = 1, \ldots, N$

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$$\mathrm{eff}_D(\xi_N) = 100 \times \left(\frac{\det(\mathbf{M}_{\xi_N, \mathbf{0}}/N)}{D_{\mathsf{opt}}} \right)^{1/p}$$

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- D_{opt} for ME and ME + 2FI models: GGHS

The question of replication

Given

- maximum number of respondents T
- q choice questions per respondent
- exact base design ξ_N of size $N < qT$

How to replicate ξ_N ?

Approaches to replication

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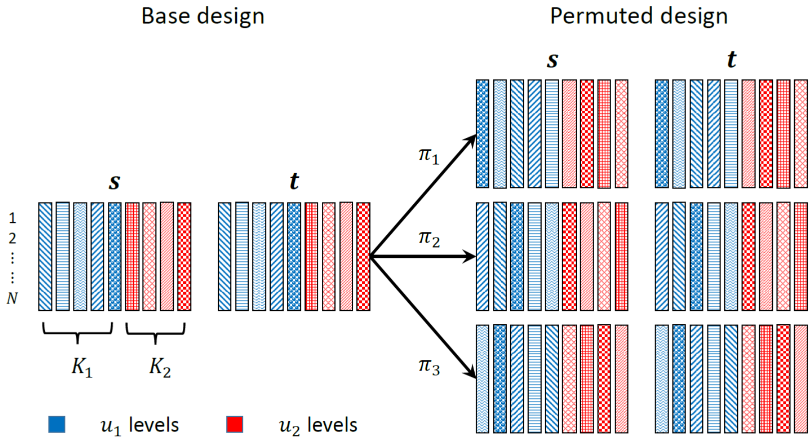
Illustration of factor permutation



Properties of factor permutation

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- $\mathbf{M}_{\xi_N,\mathbf{0}}$ block diagonal $\Rightarrow$ $\mathbf{M}_{\xi_{RN},\mathbf{0}}$ block diagonal

Example for simulation study

Entscheidung 1 von 10:

	Operation A	Operation B
Wahrscheinlichkeit für dauerhafte Inkontinenz	0 von 100 Personen	1 von 100 Personen
Häufigkeit des Wasserlassens in der Nacht	**1** Mal pro Nacht	**3** Mal pro Nacht
Dringlichkeit des Wasserlassens	**Sofort** auf Toilette müssen	**30** Minuten einhalten können
Dauer des Wasserlassens	**6** Minuten	**3** Minuten
Wahrscheinlichkeit einer erneuten Operation	20 von 100 Personen	10 von 100 Personen
Würden Sie Operation A oder B wählen?	◌	◌

Entscheidung 2 von 10:

	Operation A	Operation B
Wahrscheinlichkeit für dauerhafte Inkontinenz	0 von 100 Personen	5 von 100 Personen
Häufigkeit des Wasserlassens in der Nacht	**3** Mal pro Nacht	**5** Mal pro Nacht
Veränderung der Erektionsfähigkeit	Nimmt sehr gering ab	Unverändert
Wahrscheinlichkeit einer erneuten Operation	10 von 100 Personen	0 von 100 Personen
Wahrscheinlichkeit für eine Funktionsstörung des Samenergusses	65 von 100 Personen	5 von 100 Personen
Würden Sie Operation A oder B wählen?	◌	◌

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  - **A** D-opt. (GGS09, $N = 528$ pairs, D-eff: 100%), 3 permuted reps
  - **B** D-eff. (JMP, $N = 44$ pairs, D-eff: $\approx 77\%$), 36 non-permuted reps
  - **C** B, 36 permuted reps

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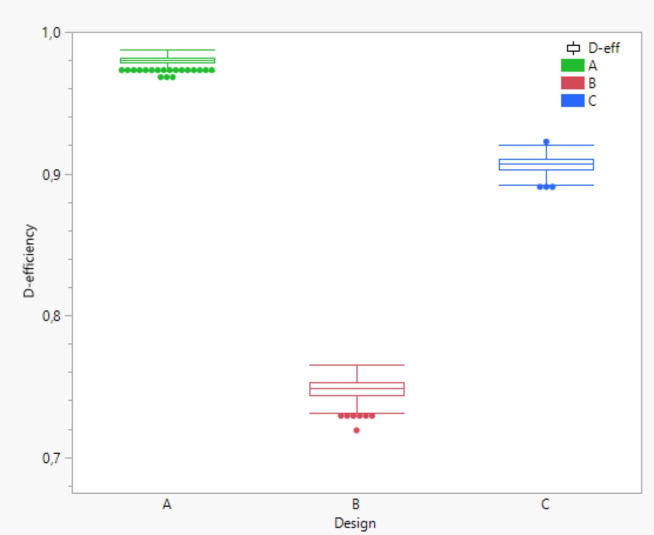
  - **C** B, 36 permuted reps
- Simulation of 1000 surveys
  - Per survey: $200/4 = 50$ respondents with 8 pairs each
  - For each of A, B, C and each survey: realized design with 400 pairs
  - Compare D-efficiencies of 1000 realized designs

Results



Conclusions

- When applicable, replication with factor permutation preferable to simple repetition of base design
- Factor permutation increases D-efficiency
- Permutation approach works well with both analytical and algorithmic D-optimal or D-efficient base designs
- Suggestion for algorithmic designs: Use larger number of choice sets than "usual"

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